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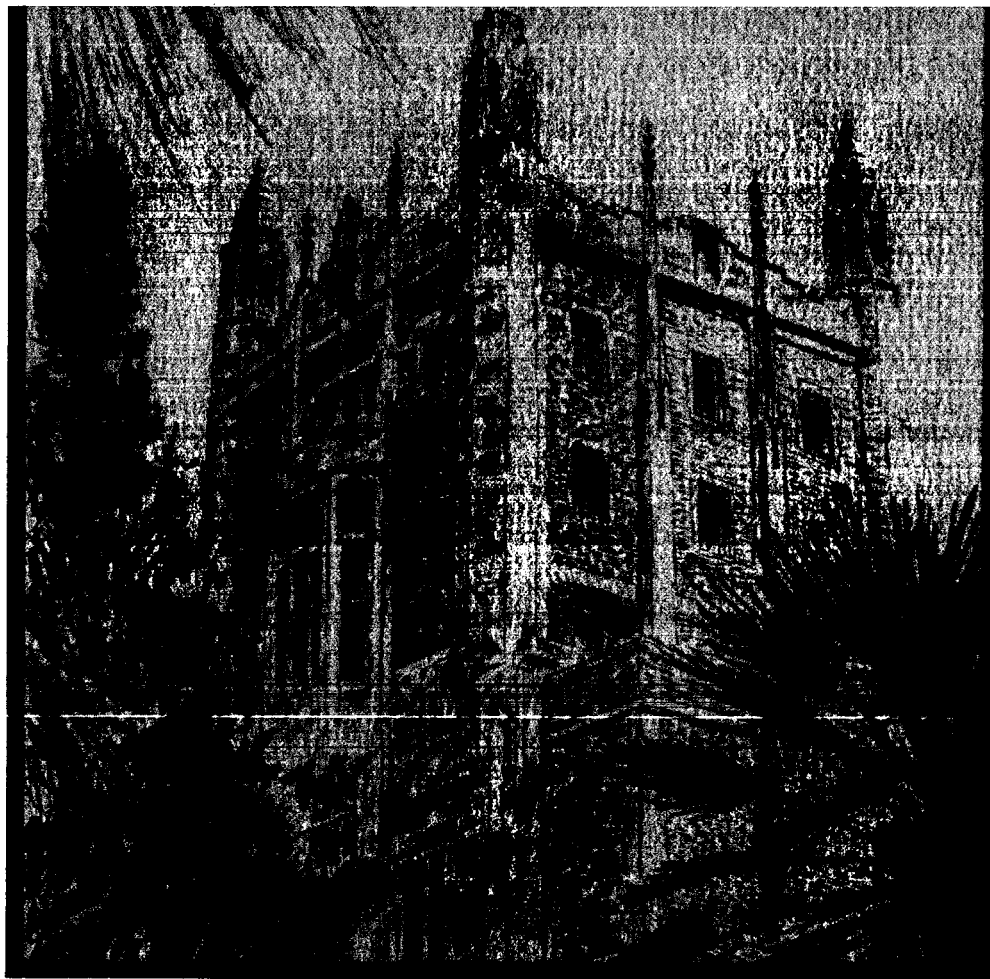
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## THE INFLUENCE OF CONVECTION THEORY ON THE FREQUENCIES OF SOLAR OSCILLATIONS

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# The Influence of Convection Theory On The Frequencies of Solar Oscillations

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## Abstract

We have studied the influence of convective theory uncertainties on the frequencies of solar oscillations. We find that the turbulent pressure plays an important role in setting the high  $\lambda$  frequencies and a lesser role in setting the low and intermediate  $\lambda$  frequencies. The representation of the turbulent pressure thus is important to the interpretation of the comparison between theory and observation. This uncertainty has not previously been recognized and may account for some of the differences between the results of various investigators since the exact value of the turbulent pressure was previously thought to have minor importance. The uncertainties in the observed frequencies of solar oscillations are too large at present to permit us to reach any conclusions about the structure of the solar envelope.

## I. Introduction

We have studied the effect on the derived frequencies of the solar oscillations of using an alternate theory of convection to calculate the thermal structure of the solar envelope. The convective theory normally used to calculate the structure of stellar envelopes is the mixing length theory. This theory is not fundamentally sound so that the derived structure is uncertain. Ideally, we would use the correct theory of convection to derive the proper structure in order to eliminate this uncertainty. Unfortunately, no correct theory of convection is available and our only way of evaluating the effect of convective theory is to use a different convection theory and hope that the difference between the derived frequencies is indicative of the uncertainty induced by convection. To this end we compare the standard mixing length theory results to the results using the non-local mixing length theory described by Ulrich (1970a, 1976). This theory is an improvement over the standard theory in that it includes the effects of the variation in the temperature gradient over the distance of a mixing length and derives the turbulent pressure and convective flux from a similar non-local average. This theory nonetheless contains arbitrary parameters and assumptions like the standard theory and is not fundamentally sound. It is nonetheless distinct from a variety of local theories such as those due to Spiegel (1963), Unno (1967) and Gough (1977) which end up with a formula which relates the convective flux at a point in the envelope to the temperature gradient at that same point. The last theory was primarily aimed at the development of a time dependent treatment. Local theories as applied to a static envelope differ in terms of the specific relationships between the variables but cannot include the global coupling found in the non-local theory. As long as the solar models deduced from the local theories satisfy the constraints of luminosity and radius defining the sun, the variations in detailed structure produced by these differences are more restricted in character than the changes produced by the non-local theory.

We find two effects from using the non-local theory - first, the sound velocity is increased just below the surface due to a steeper temperature gradient and second, the density and pressure scale heights are lengthened in the low photosphere due to turbulent pressure from convective overshoot. The second effect can also be induced in the local mixing length theory by increasing the ratio of turbulent pressure to superadiabatic temperature gradient. This second effect dominates the frequency changes and causes the frequencies to decrease when the non-local theory is used or when the turbulent pressure is increased. The changes are greatest for the highest  $\lambda$  values and also largest for the higher frequencies.

Convection theory applied to stellar atmosphere structure contains several arbitrary parameters which are normally related to a quantity called the mixing length. The atmosphere structure equations require knowledge of the convective flux and the turbulent pressure. Both quantities are derived from the products of fluctuations associated with vertical motions in the atmosphere. The flux is proportional to  $\rho \langle v_r H' \rangle$  while the turbulent pressure is proportional to  $\rho \langle v_r^2 \rangle$ . The quantity  $H'$  in the flux expression is the fluctuation in the enthalpy perturbation which is approximately equal to  $C_p T'$ ;  $C_p$  is the heat capacity at constant pressure and  $T'$  is the temperature fluctuation. Local mixing length theory assumes that  $\nabla - \nabla_{ad}$ , the difference between the actual average temperature gradient  $d \log T / d \log P$  and the adiabatic temperature gradient  $(\partial \log T / \partial \log P)_S$ , is constant over a pressure scale height. The averages in the expressions for the flux and turbulent pressure are then calculated using some model for the motion of the convecting fluid. Usually this model takes the very simple form of an assumption that a mass of fluid begins motion from a stationary condition, accelerates unimpeded for a distance equal to the mixing length and then abruptly disappears. Clearly the true state of a stellar atmosphere is more complex than this assumption. We focus our attention on just two inadequacies of the mixing length theory:

- 1) The effective distance of travel involved in the average for  $\langle v_r H' \rangle$  need not be the same as the distance of travel for  $\langle v_r^2 \rangle$ .
- 2) The temperature gradient  $\nabla - \nabla_{ad}$  changes significantly over a pressure scale height near the point where the atmosphere becomes convectively unstable.

Although mixing length theory is inadequate for numerous reasons, we concentrate on the above two points because they are amenable to study with our presently available numerical techniques.

### III. The Turbulent Pressure

In the context of mixing length theory we can define the point of origin of a mass of fluid as that point in space and time when it has a zero average vertical velocity. This mass accelerates, moves a distance  $\pm l$  in the vertical direction and then loses its coherence. The theory which results may be written in the form:

$$\frac{\langle v_r^2 \rangle}{v_s^2} = \alpha_v^2 (\nabla - \nabla_{ad}) \quad (1)$$

$$\frac{\langle v_r H' \rangle}{v_s C_p T} = \alpha_F^2 (\nabla - \nabla_{ad})^{3/2} \quad (2)$$

where  $\alpha_v$  and  $\alpha_F$  are roughly to be identified as the effective ratios of mixing length to pressure scale height for the turbulent velocity and convective flux respectively. The sound speed is  $v_s$ . The quantities  $\alpha_v$  and  $\alpha_F$  normally contain additional factors of order unity which account for known thermodynamic derivatives, radiative heat exchange between the fluid mass and its surroundings and other convection model dependent factors. For the purposes of the exposition here we neglect these factors. The actual calculations were made following the formulation described by Henyey, Vardya and Bodenheimer (1965) with a few minor modifications described by Ulrich (1970 a,b).

When a model atmosphere is calculated, the structure equations require that essentially all of the flux be carried by convection throughout the unstable layers except for a very thin zone just below the point of marginal stability. Consequently, equation (2) fixes  $(\nabla - \nabla_{ad})$ .

The turbulent pressure  $p \langle v_r^2 \rangle$  is then fixed to within a factor of  $(\alpha_v / \alpha_F)^2$ . Because the velocity and enthalpy fluctuations need not follow the same pattern of growth during the motion of the mass of fluid, this ratio may differ from the value given by the standard mixing length theory. In order to study the effect of this uncertainty, we have multiplied the turbulent pressure by an additional factor  $\beta$ . When  $\beta = 1$  we recover the mixing length theory of Henyey et al (1965).

#### IV. Non-Local Effects

The second uncertainty we have studied involves the effects of variations with depth of  $\nabla - \nabla_{ad}$  on the local averages  $\langle v_r^2 \rangle$  and  $\langle v_r H' \rangle$ . The theory described by Ulrich (1970c) provides a method for deriving each of these averages from an integral over the parameters throughout a nearby region of the atmosphere. The algorithm for computing the structure iteratively corrects the temperature distribution until the non-local averages produce a convective flux which is equal to the total flux minus the radiative flux throughout the convective envelope. Because of the extended and complicated coupling which this formulation provides, convergence of the algorithm is slow and usually produces flux constancy to within 2 to 3 percent. The final model is nonetheless significantly different from the local convection model and is well enough defined that meaningful comparisons are possible. The principal differences between the non-local and the local models are:

- 1) The location of the maximum in  $\nabla - \nabla_{ad}$  is displaced inward by about one half a pressure scale height and the value of the maximum in  $\nabla - \nabla_{ad}$  is roughly doubled.
- 2) The nominally stable region just outside the point of marginal stability has non-zero  $\langle v_r^2 \rangle$  because of convective overshoot.

The overshoot turbulent pressure is similar to the uncertainty in  $\langle v_r^2 \rangle$  discussed above in the context of the local convective theory.

#### V. The Models

We have identified two areas of uncertainty in the convective theory describing the solar envelope: 1) The ratio of turbulent pressure to convective flux and 2) The detailed temperature gradient near the point of marginal convective stability. In order to study these two effects we have computed three solar models which have characteristics given in Table 1.

TABLE 1  
SOLAR MODELS

Model 1*	Standard solar model - local convective theory and $\beta = 1$ .
Model 2	Local convective theory model with $\beta = 1.5$ .
Model 3	Non-local convection theory model using the Ulrich (1970c) iterative procedure.

\* These models have UCLA reference numbers of 22L, 38, and 41 respectively. The previous standard model was 22C. This model did not have proper consistency between equations 1 and 2. This inconsistency which has now been corrected does not appear to alter any of our previous conclusions based on model 22C. The UCLA reference numbers will not be used elsewhere in this report.

The structural changes produced by altering the value of  $\beta$  can be seen by comparing Models 1 and 2. Figure 1 shows this comparison in the two right hand panels. Each panel shows differences between the models which have been interpolated so that points at equal distances below the layer with optical depth unity are compared. These differences are then plotted as functions of the depth below this same zero point. The change in  $\beta$  displaces the run of physical variables so that the model with smaller  $\beta$  has larger temperature and density. The change is smooth but involves essentially all the convection zone. In contrast, the use of the non-local convection theory produces complex changes near the surface where the detailed temperature and density stratification is redistributed. The overshoot process alters the structure in the optically thin layers which were unchanged by the turbulent pressure variation. Models 2 and 3 are compared rather than 1 and 3 since Model 2 matches Model 3 more closely in the deeper parts of the convection zone and thus provides a more direct indication of the specific influence of the non-local convection theory. The remaining change in the asymptotic behavior is nonetheless similar to the change brought about by a change in  $\beta$  although of opposite sign. Consequently, the two effects are combined in the comparison of Models 2 and 3. Note in particular that the density difference between Models 2 and 3 is larger than the difference between Models 1 and 2.

## VI. The Frequencies and Observations

Figure 2 shows the increase in frequency caused by decreasing  $\beta$  from 1.5 to 1.0 on the left. On the right Figure 2 shows the increase in frequency caused by using the local convection theory instead of the non-local theory. At least part of this change is a result of the higher density and temperature in the deeper parts of the envelope of Model 2 as compared to Model 3. Note however that at the higher values of  $\nu$  and  $\lambda$  the pattern of frequency change differs between the two parts of Figure 2. This pattern difference in principle can be used to distinguish between the effect of  $\beta$  and the effect of the non-local theory. For values of  $\lambda$  less than 100 the frequency shifts were smaller than those shown in Figure 2.

The available observations fall into three groups: first, the global oscillations which have been measured by Claverie, et al (1979), Grec, Fossat and Pomerantz (1980) and Woodard (1984); second, the intermediate degree modes which have been measured by Duvall and Harvey (1983), and Harvey and Duvall (1984); and the high degree modes which have been measured by Deubner, Ulrich and Rhodes (1979) and Deubner (1983). The global modes have the most precisely measured frequencies and the high degree modes the least precisely measured frequencies. Unfortunately, the low degree modes are not able to distinguish between the possible convection models and the high degree frequencies are not precise enough to provide clear evidence. In fact even for the intermediate degree modes the uncertainty in the frequencies given by Harvey and Duvall (1984) for the modes with  $\lambda$  above about 40  $\mu\text{Hz}$  is 10  $\mu\text{Hz}$  and this uncertainty is large enough to prevent us from drawing definite conclusions. Figures 3 to 5 show the comparison between the theory and observations. Model 1 is in reasonably good agreement with Harvey and Duvall (1984) although the erratic trend in  $\Delta\nu$  indicates that either the models or the observations are inaccurate. The errors in the frequencies for the high  $\lambda$  modes are clearly too large to permit a discrimination between the models. Only Model 3 appears to be noticeably less satisfactory than the others.

In summary, we draw three conclusions:

- 1) The representation of the turbulent pressure is an important part of mixing length theory.
- 2) The difference between the effects of turbulent pressure and the non-local convection theory can be measured at high  $\lambda$ .
- 3) The low  $\lambda$  mode frequencies are only slightly dependent on the convection zone uncertainties.
- 4) Present frequency measurements are inadequate to permit us to reach any conclusions concerning the properties of the solar convection zone.

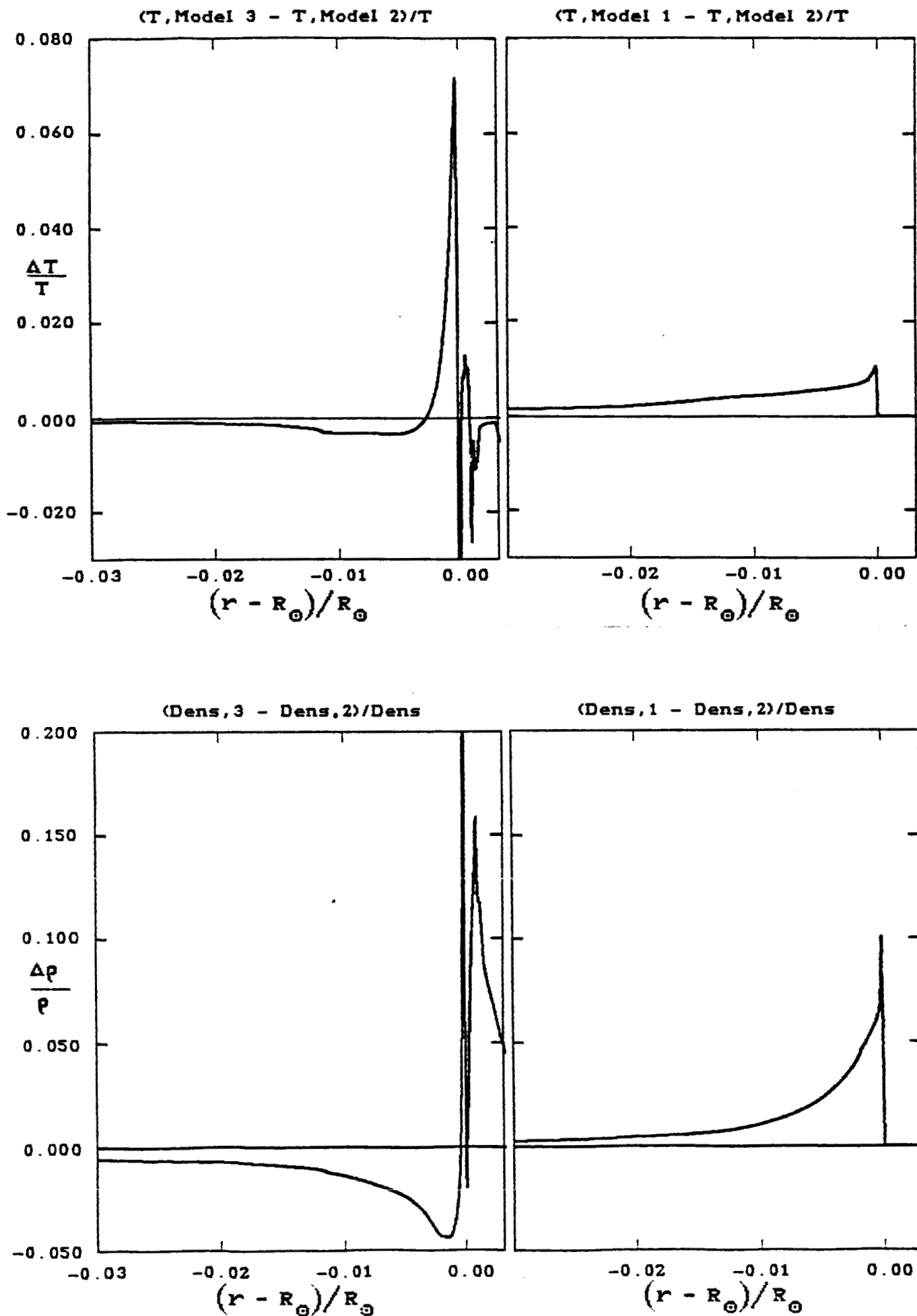


Figure 1 Comparison of the structure of the three solar models discussed in the text. Each figure shows the fractional differences between pairs of models as indicated.

Model 1 - Model 2

Model 2 - Model 3

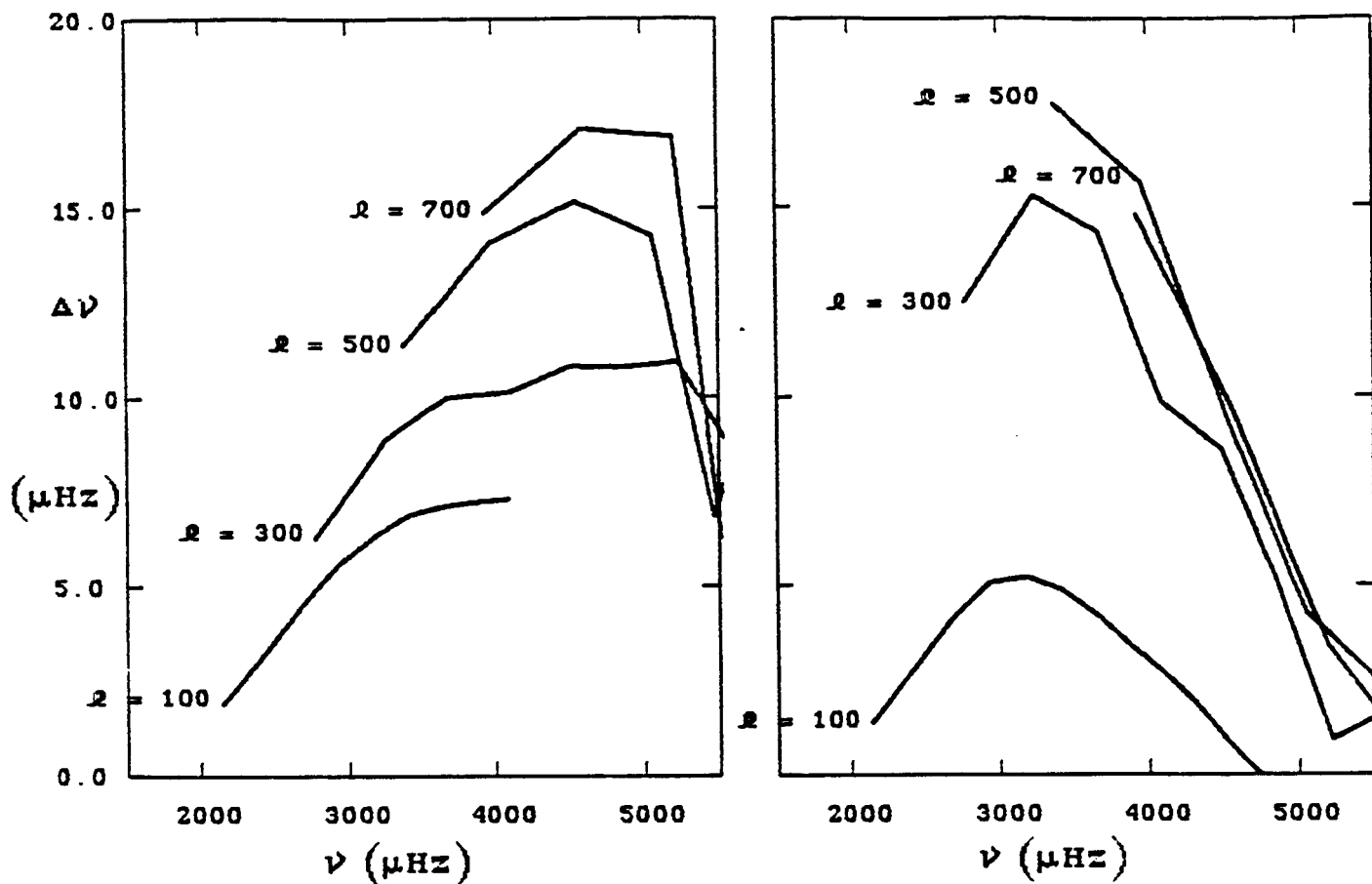


Figure 2 Comparison of frequencies deduced for the three solar models.

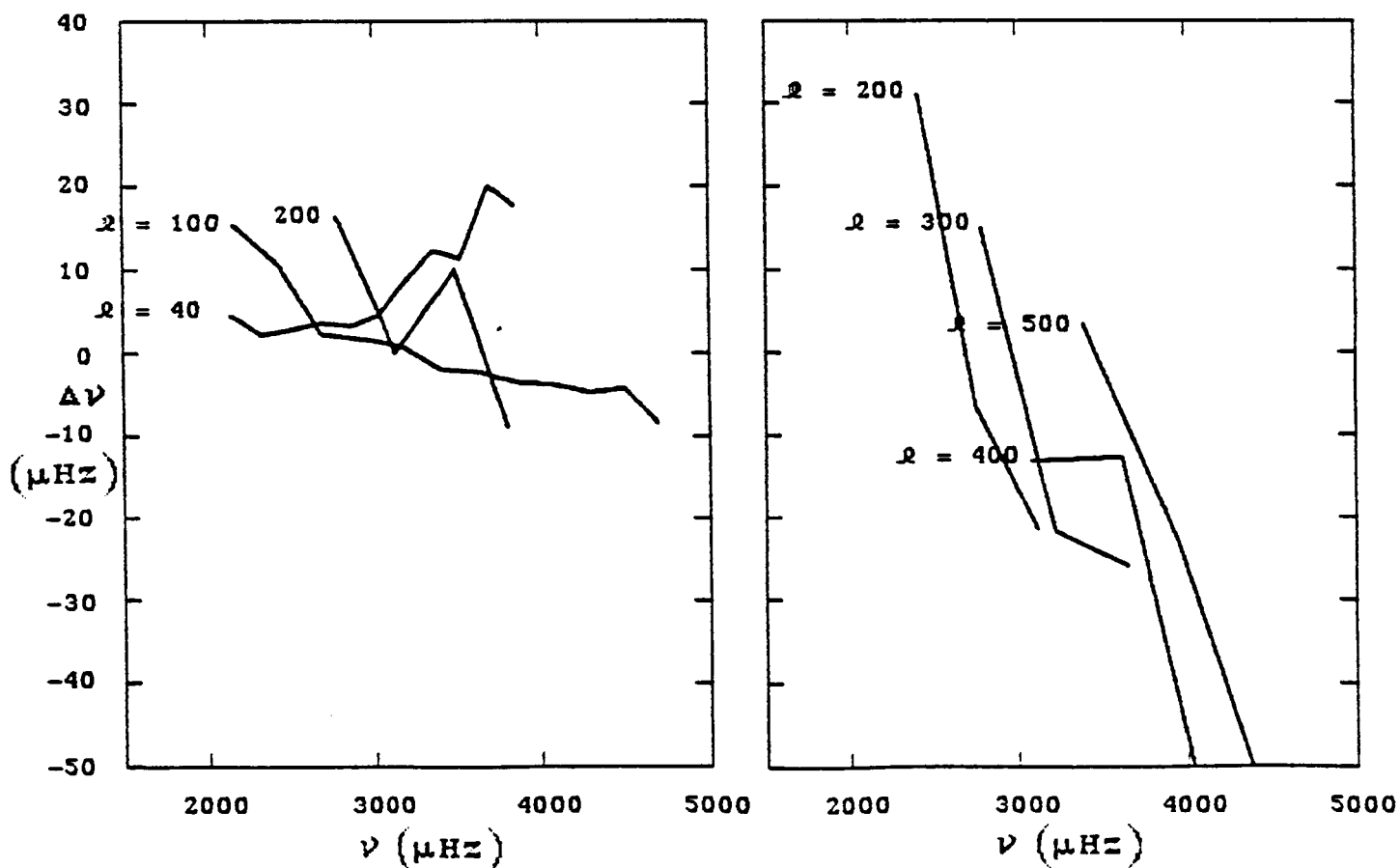


Figure 3 Comparison of the frequencies for Model 1 to observations. The observations are from Duvall and Harvey, 1984 on the left and from Deubner, Ulrich and Rhodes, 1979 on the right.

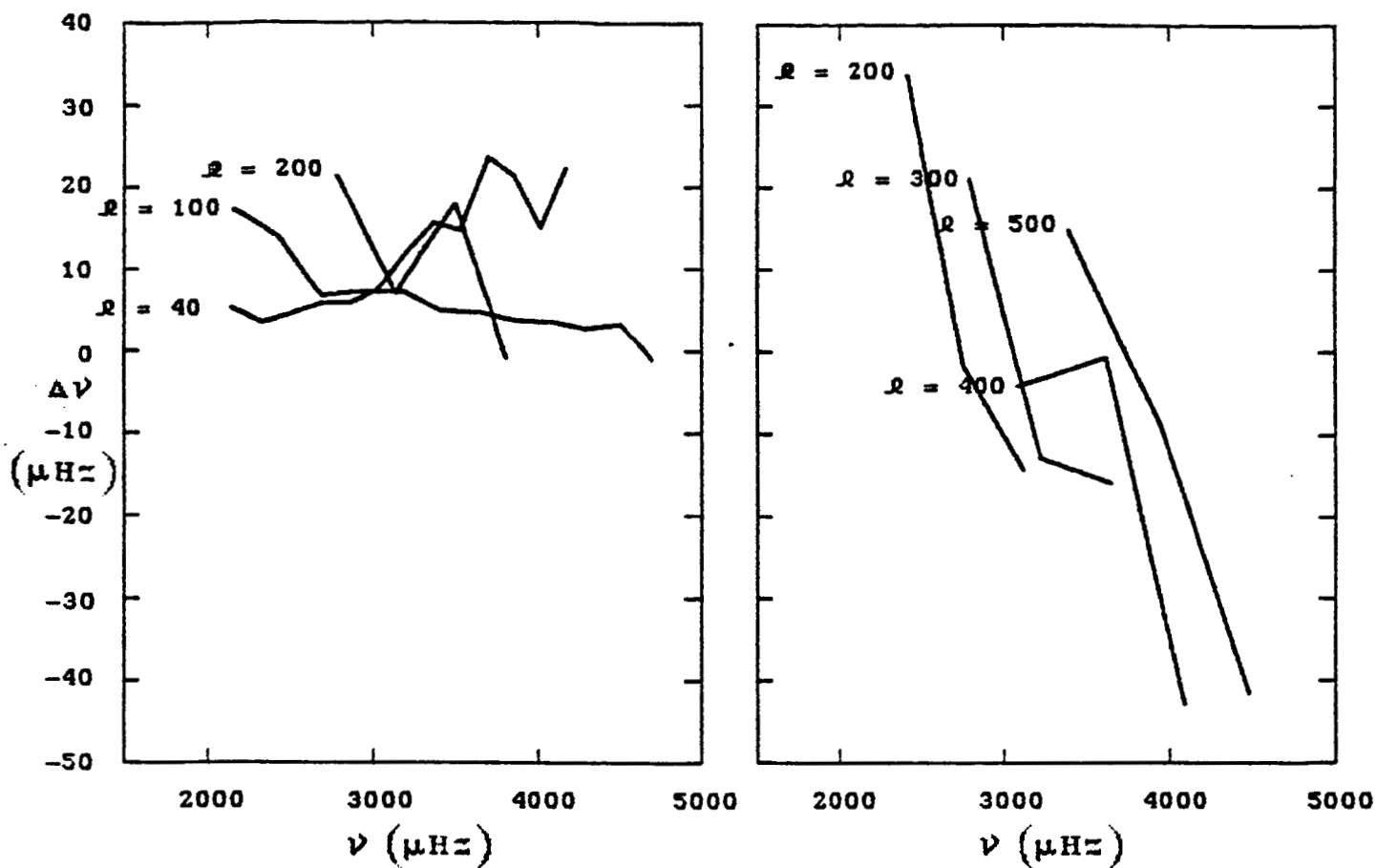


Figure 4 Comparison of the frequencies for Model 2 to observations from Duvall and Harvey, 1984 on the left and from Deubner, Ulrich and Rhodes, 1979 on the right.

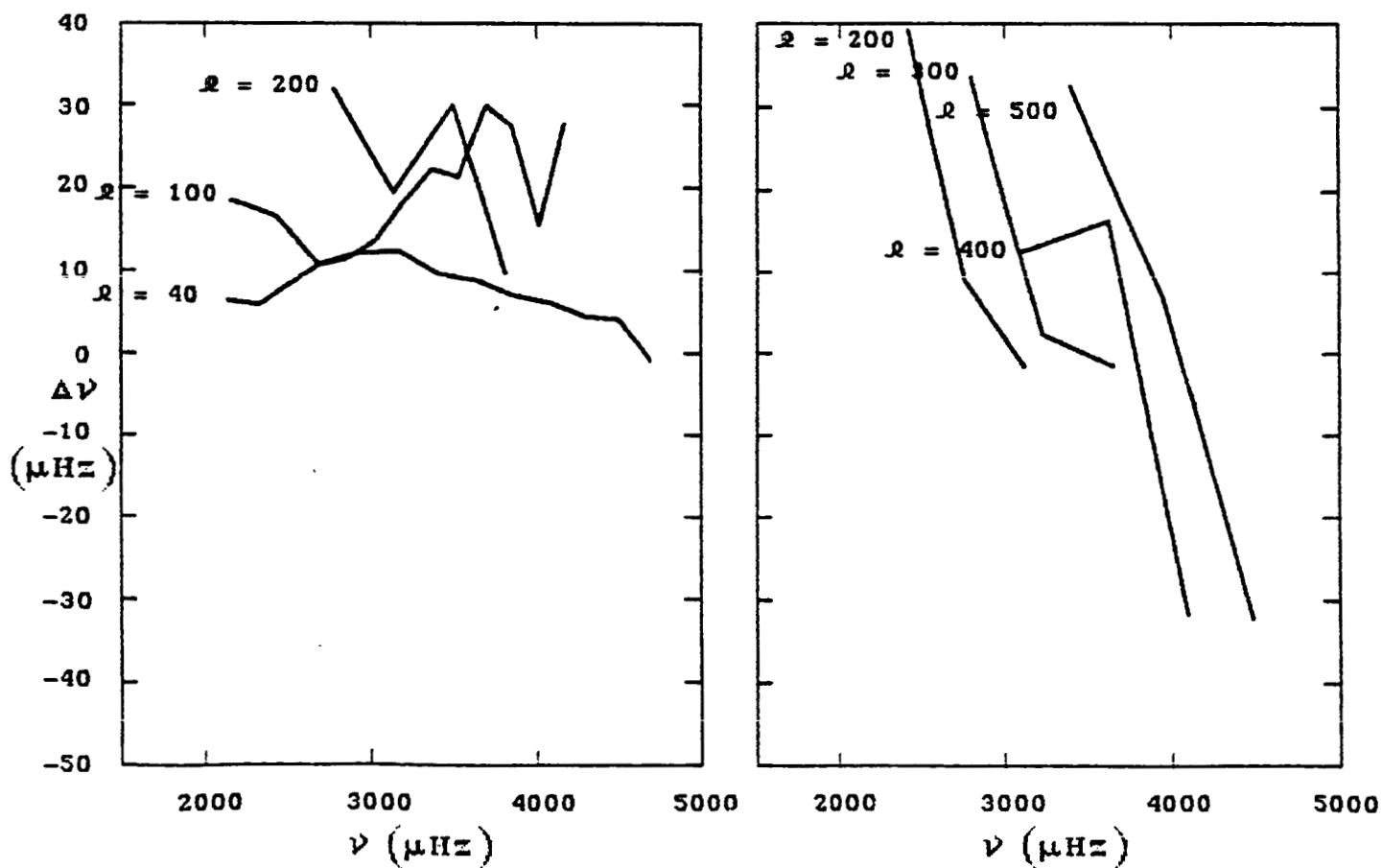


Figure 5 Comparison of the frequencies for Model 3 to observations from Duvall and Harvey, 1984 on the left and from Deubner, Ulrich and Rhodes, 1979 on the right.



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